Probability density function model equation for particle charging in a homogeneous dusty plasma

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In this paper, we use the direct interaction approximation (DIA) to obtain an approximate integrodifferential equation for the probability density function (PDF) of charge (q) on dust particles in homogeneous dusty plasma. The DIA is used to solve the closure problem which appears in the PDF equation due to the interactions between the phase space density of plasma particles and the phase space density of dust particles. The equation simplifies to a differential form under the condition when the fluctuations in phase space density for plasma particles change very rapidly in time and is correlated for very short times. The result is a Fokker-Planck type equation with extra terms having third and fourth order differentials in q, which account for the discrete nature of distribution of plasma particles and the interaction between fluctuations. Approximate macroscopic equations for the time evolution of the average charge and the higher order moments of the fluctuations in charge on the dust particles are obtained from the differential PDF equation. These equations are computed, in the case of a Maxwellian plasma, to show the effect of density fluctuations of plasma particles on the statistics of dust charge.

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I. INTRODUCTION

The phenomenon of a dusty plasma creates a physical situation in which nanoscale (dust) particles are formed from molecular species and acquire charge by interacting with electrons and ions (hereafter referred to as ''plasma particles''). In recent years, through experimental studies [1–3] in radio-frequency plasma reactors, a general understanding of the phenomenon has evolved into a ''four-step'' theoretical model describing the birth and growth of dust particles and their effect on the plasma parameters [4] (see also Refs. [5,6] for recent reviews). These four steps include the following

(i) The generation of supersmall $[O(2\ \mathrm{nm})]$ particles from molecular species.

(ii) The charging and selective trapping or levitation of supersmall or dust particles.

(iii) The growth of nanoparticles due to coagulation of dust particles.

(iv) The α-γ' transition phenomenon during the coagulation process when the radius of the dust particle becomes higher than a critical value and the electron losses on the particle become more essential than those on the walls of the reactor. In this situation, the electron concentration decreases dramatically and, consequently, the electron temperature increases to support the plasma balance in the reactor [4].

The dust particles (hereafter simply referred to as ''particles'') move under the influence of forces that are stochastic in nature, and the charging mechanism, collision, and coagulation of the particles further enhance the complexity to develop a predictive theory from first principles [7–9] for the description of the phenomenon.

In this work, we restrict our attention to the charging of the particles in homogeneous plasma, without taking into account their back effects on the plasma particle distribution and the related plasma parameters. We assume the particle velocity to be negligible in comparison to the plasma particle velocity. This ideal situation is identical to the situation studied in the recent past [10–17]. These studies further provided the foundation for investigations on particle growth [18–21] and heating of particles [22,23].

When the charge of the plasma particles (e_σ) is small, the equation for the phase space density $f(q,t)$ for the particle charge q at time t is given by [9]

$$\frac{\partial}{\partial t} f(q,t) + \frac{\partial}{\partial q}[I f(q,t)] - \frac{\partial^2}{\partial q^2}[Q f(q,t)] = 0, \quad (1.1)$$

where

$$I = \sum_\sigma \int e_\sigma \gamma_\sigma \upsilon_\sigma f_\sigma(\mathbf{r}, \mathbf{v}_\sigma, t) d\mathbf{v}_\sigma \quad (1.2)$$

and

$$Q = \frac{1}{2} \sum_\sigma \int e_\sigma^2 \gamma_\sigma \upsilon_\sigma f_\sigma(\mathbf{r}, \mathbf{v}_\sigma, t) d\mathbf{v}_\sigma, \quad (1.3)$$

with boldface indicating a vector. Here the subscript $\sigma = \{i,e\}$ represents properties for ions (i) and electrons (e), and $f_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)$ represents the phase space density for plasma particles, with $\mathbf{r}$ and $\mathbf{v}_\sigma$ denoting the position and velocity of plasma particles, respectively. We assume the velocity of the particles ($\mathbf{v}$) to be negligible in comparison to the plasma particle velocity ($\mathbf{v}_\sigma$), and write $\upsilon_\sigma = |\mathbf{v}_\sigma - \mathbf{v}| \cong |\mathbf{v}_\sigma|$. Here γ_σ is a cross section for charging collisions between dust particles and plasma particles, and is determined by the orbit motion limited approach [24],

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$$\gamma_\sigma = \pi a^2 \left(1 - \frac{2 e_\sigma q}{4\pi\epsilon_0 a m_\sigma v_\sigma^2}\right) \Theta\left(1 - \frac{2 e_\sigma q}{4\pi\epsilon_0 a m_\sigma v_\sigma^2}\right), \tag{1.4}$$

where a is the particle radius, m_σ is the mass of plasma particle, ϵ_0 is the permittivity, and Θ is the Heaviside step function:

$$\Theta(x) = \begin{cases} 1, & x \geqslant 0 \\ 0, & x < 0. \end{cases}$$

The function f_σ in Eqs. (1.2) and (1.3) is stochastic in nature, thus also making $f(q,t)$ in Eq. (1.1) a stochastic function. The equation for the probability density function (PDF) can now be obtained by taking the ensemble average of Eq. (1.1) over a large number of realizations, and then normalizing it by the total number of dust particles (n_0). We denote the ensemble average by $\langle\ \rangle$ and define

$$f(q,t) = n_0 \overline{f}(q,t) + \tilde{f}(q,t), \quad n_0 \overline{f} = \langle f \rangle, \quad \langle \tilde{f} \rangle = 0, \tag{1.5}$$

$$f_\sigma(\mathbf{r},\mathbf{v}_\sigma,t) = \overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t) + \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t),$$
$$\overline{f}_\sigma = \langle f_\sigma \rangle, \quad \langle \tilde{f}_\sigma \rangle = 0, \tag{1.6}$$

$$I = \overline{I} + \tilde{I}, \quad \overline{I} = \langle I \rangle = \sum_\sigma \int e_\sigma \gamma_\sigma v_\sigma \overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t) d\mathbf{v}_\sigma, \tag{1.7}$$

$$\langle \tilde{I} \rangle = 0,$$

$$Q = \overline{Q} + \tilde{Q},$$

$$\overline{Q} = \langle Q \rangle = \frac{1}{2} \sum_\sigma \int e_\sigma^2 \gamma_\sigma v_\sigma \overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t) d\mathbf{v}_\sigma, \quad \langle \tilde{Q} \rangle = 0. \tag{1.8}$$

Note that $\overline{f}$ is defined such that it satisfies

$$\int \overline{f}(p,t) dp = 1, \tag{1.9}$$

and, therefore, it is a probability density function. Now, the ensemble average of Eq. (1.1) over a large number of realizations yields

$$\frac{\partial}{\partial t}\overline{f}(q,t) + \frac{\partial}{\partial q}[\overline{I}\,\overline{f}(q,t)] - \frac{\partial^2}{\partial q^2}[\overline{Q}\,\overline{f}(q,t)]$$
$$= -\frac{\lambda}{n_0}\frac{\partial}{\partial q}[\langle \tilde{I} f(q,t)\rangle] + \frac{\lambda}{n_0}\frac{\partial^2}{\partial q^2}[\langle \tilde{Q} f(q,t)\rangle], \tag{1.10}$$

having unknown correlations $\langle \tilde{I} f \rangle$ and $\langle \tilde{Q} f \rangle$ which pose a problem of closure similar to well-known turbulence closure problem. In Eq. (1.10), we have introduced λ, a usual perturbation expansion parameter, for future convenience and which is set equal to one at the end of our calculations later in this paper.

Equation (1.10), with $\lambda = 0$, was obtained by Matsouka and co-workers [11–13] from the master equation for PDF ($\overline{f}$), written in discrete form and for which a solution was given by assuming two different expressions for $\overline{f}_\sigma$ describing Maxwellian and Druyvesteyn distributions for the energy of plasma particles. Later, Gordiets and Ferreira [14,15] extended these works to include secondary electron emission charging mechanism in addition to the absorption of plasma particles described by Eq. (1.7) and (1.8). A situation with different charging mechanisms was further analyzed within the Langevin approach by Vaulina *et al.* [17] and Khrapak *et al.* [16].

In this paper we do *not* ignore interactions between fluctuations which cause the appearance of unknown correlations in Eq. (1.10) while considering charging only due to the absorption of plasma particles. We obtain expressions for these correlations by using the direct interaction approximation (DIA), which was proposed by Kraichnan in his pioneering work [25] as a renormalized perturbation method to solve turbulence closure problem. This leads to a closed set of integrodifferential equations for PDF ($\overline{f}$). We then simplify the equations, under certain conditions, and obtain an approximate differential equation for the PDF $\overline{f}$, from which macroscopic equations [26] governing the temporal evolution of the average of q and its higher moments are derived. These macroscopic equations are computed to obtain the temporal evolution of the average of q and its variance, and the steady state values for the skewness and kurtosis for a particular case of Maxwellian plasma. These results exhibit the effects on the statistics of the dust charge due to the fluctuations-fluctuation interactions neglected in the previous studies [11–13,8,9].

II. DIA CLOSURE EQUATIONS FOR THE PDF AND THE GREEN'S FUNCTION

In this section, we apply the DIA [25,27] method of closure to obtain approximate expressions for unknown correlations $\langle \tilde{I} f \rangle$ and $\langle \tilde{Q} f \rangle$. Equation (1.1) is linear in f for a prescribed statistical description for f_σ which does not depend on f, and then we have

$$f(q,t) = \int \hat{G}(q,t;p,t_0) f(p,t_0) dp, \tag{2.1}$$

where $\hat{G}(q,t;p,t')$ is the Green's function that satisfies $\forall\, t > t'$:

$$\frac{\partial}{\partial t}\hat{G}(q,t;p,t') + \frac{\partial}{\partial q}[\overline{I}\hat{G}(q,t;p,t')] - \frac{\partial^2}{\partial q^2}[\overline{Q}\hat{G}(q,t;p,t')]$$
$$= -\lambda\frac{\partial}{\partial q}[\tilde{I}\hat{G}(q,t;p,t')] + \lambda\frac{\partial^2}{\partial q^2}[\tilde{Q}\hat{G}(q,t;p,t')], \tag{2.2}$$

$$\hat{G}(q,t';p,t')=\delta(q-p). \tag{2.3}$$

Since $\hat{G}(q,t;p,t')$ is not a functional of f and is statistically independent of the initial f at time t_0, we can write

$$\overline{f}(q,t)=\int G(q,t;p,t_0)\overline{f}(p,t_0)dp, \tag{2.4}$$

where

$$G(q,t;p,t')=\langle \hat{G}(q,t;p,t')\rangle, \tag{2.5}$$

and thus PDF $\overline{f}(q,t)$ can be completely determined by the average Green's function G.

We proceed by ensemble averaging Eq. (2.2) to obtain

$$\frac{\partial}{\partial t}G(q,t;p,t')+\frac{\partial}{\partial q}[\overline{I}G(q,t;p,t')]-\frac{\partial^2}{\partial q^2}[\overline{Q}G(q,t;p,t')]$$
$$=-\lambda\frac{\partial}{\partial q}[\langle \tilde{I}\hat{G}(q,t;p,t')\rangle]+\lambda\frac{\partial^2}{\partial q^2}[\langle \tilde{Q}\hat{G}(q,t;p,t')\rangle]. \tag{2.6}$$

Expanding $\hat{G}$ in perturbation series in λ as

$$\hat{G}=G_0+\lambda\hat{G}_1+\lambda^2\hat{G}_2+\cdots, \tag{2.7}$$

substituting it in Eq. (2.2), and equating the terms with equal powers in λ, yields equations for $G_0,\hat{G}_n\{n=1,2,\ldots\}$, written as

$$\frac{\partial}{\partial t}G_0(q,t;p,t')+\frac{\partial}{\partial q}[\overline{I}G_0(q,t;p,t')]$$
$$-\frac{\partial^2}{\partial q^2}[\overline{Q}G_0(q,t;p,t')]=0, \tag{2.8}$$

$$\frac{\partial}{\partial t}\hat{G}_n(q,t;p,t')+\frac{\partial}{\partial q}[\overline{I}\hat{G}_n(q,t;p,t')]$$
$$-\frac{\partial^2}{\partial q^2}[\overline{Q}\hat{G}_n(q,t;p,t')]$$
$$=-\frac{\partial}{\partial q}[\tilde{I}\hat{G}_{n-1}(q,t;p,t')]+\frac{\partial^2}{\partial q^2}[\tilde{Q}\hat{G}_{n-1}(q,t;p,t')]. \tag{2.9}$$

Equation (2.8) suggests that G_0 is a statistically sharp function and the solution for G_n from Eqs. (2.8) and (2.9) can be written as

$$\hat{G}_n(q,t;p,t')=\int_{t'}^{t}ds\int G_0(q,t;z,s)$$
$$\times\left\{-\frac{\partial}{\partial z}[\tilde{I}(z,s)\hat{G}_{n-1}(z,s;p,t')]\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}[\tilde{Q}(z,s)\hat{G}_{n-1}(z,s;p,t')]\right\}dz. \tag{2.10}$$

Using Eqs. (2.7) and (2.10), we can write $\langle\tilde{I}\hat{G}\rangle$ and $\langle\tilde{Q}\hat{G}\rangle$ up to first order in λ as

$$\langle\tilde{I}(q,t)\hat{G}(q,t;p,t')\rangle$$
$$=\lambda\int_{t'}^{t}ds\int G_0(q,t;z,s)$$
$$\times\left\{-\frac{\partial}{\partial z}[\langle\tilde{I}(q,t)\tilde{I}(z,s)\rangle G_0(z,s;p,t')]\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}[\langle\tilde{I}(q,t)\tilde{Q}(z,s)\rangle G_0(z,s;p,t')]\right\}dz, \tag{2.11}$$

$$\langle\tilde{Q}(q,t)\hat{G}(q,t;p,t')\rangle$$
$$=\lambda\int_{t'}^{t}ds\int G_0(q,t;z,s)$$
$$\times\left\{-\frac{\partial}{\partial z}[\langle\tilde{Q}(q,t)\tilde{I}(z,s)\rangle G_0(z,s;p,t')]\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}[\langle\tilde{Q}(q,t)\tilde{Q}(z,s)\rangle G_0(z,s;p,t')]\right\}dz. \tag{2.12}$$

The expressions for correlations $\langle\tilde{I}(q,t)\tilde{I}(q',t')\rangle$, $\langle\tilde{I}(q,t)\tilde{Q}(q',t')\rangle$, and $\langle\tilde{Q}(q,t)\tilde{Q}(q',t')\rangle$ are now written as

$$\langle\tilde{I}(q,t)\tilde{I}(q',t')\rangle=\sum_{\sigma}\int\int e_{\sigma}^{2}\gamma_{\sigma}(q,v_{\sigma})\gamma_{\sigma}(q',v'_{\sigma})v_{\sigma}v'_{\sigma}$$
$$\times\langle\tilde{f}_{\sigma}(\mathbf{r},\mathbf{v}_{\sigma},t)\tilde{f}_{\sigma}(\mathbf{r},\mathbf{v}'_{\sigma},t')\rangle d\mathbf{v}_{\sigma}d\mathbf{v}'_{\sigma}, \tag{2.13}$$

$$\langle\tilde{I}(q,t)\tilde{Q}(q',t')\rangle=\sum_{\sigma}\int\int\frac{e_{\sigma}^{3}}{2}\gamma_{\sigma}(q,v_{\sigma})\gamma_{\sigma}(q',v'_{\sigma})v_{\sigma}v'_{\sigma}$$
$$\times\langle\tilde{f}_{\sigma}(\mathbf{r},\mathbf{v}_{\sigma},t)\tilde{f}_{\sigma}(\mathbf{r},\mathbf{v}'_{\sigma},t')\rangle d\mathbf{v}_{\sigma}d\mathbf{v}'_{\sigma}, \tag{2.14}$$

and

$$\langle \tilde{Q}(q,t)\tilde{Q}(q',t')\rangle = \sum_\sigma \int\int \frac{e_\sigma^4}{4}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q',v'_\sigma)v_\sigma v'_\sigma \times\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,t')\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma. \tag{2.15}$$

While writing Eqs. (2.13), (2.14), and (2.15), we have assumed that fluctuating parts of the phase space density for ions and electrons are not correlated, i.e., $\langle \tilde{f}_i(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_e(\mathbf{r},\mathbf{v}_\sigma,t')\rangle = 0$. Substitution of various correlations, as obtained from Eqs. (2.13), (2.14), and (2.15), in (2.11) and (2.12), and replacing G_0 by G as a process of renormalization [27,28], yield, for $\lambda=1$,

$$\begin{aligned}\langle \tilde{I}(q,t)\hat{G}(q,t;p,t')\rangle &= \int_{t'}^{t} ds\int G(q,t;z,s)\Bigg\{-\frac{\partial}{\partial z}\Bigg[\sum_\sigma \int\int e_\sigma^2\gamma_\sigma(q,v_\sigma)\gamma_\sigma(z,v'_\sigma)v_\sigma v'_\sigma\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(z,s;p,t')\Bigg]\\ &\quad + \frac{\partial^2}{\partial z^2}\Bigg[\sum_\sigma \int\int \frac{e_\sigma^3}{2}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(z,v'_\sigma)v_\sigma v'_\sigma\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(z,s;p,t')\Bigg]\Bigg\} dz\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}\langle \tilde{Q}(q,t)\hat{G}(q,t;p,t')\rangle &= \int_{t'}^{t} ds\int G(q,t;z,s)\Bigg\{-\frac{\partial}{\partial z}\Bigg[\sum_\sigma \int\int \frac{e_\sigma^3}{2}\gamma_\sigma(z,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,s)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,t)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(z,s;p,t')\Bigg]\\ &\quad + \frac{\partial^2}{\partial z^2}\Bigg[\sum_\sigma \int\int \frac{e_\sigma^4}{4}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(z,v'_\sigma)v_\sigma v'_\sigma\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(z,s;p,t')\Bigg]\Bigg\} dz.\end{aligned} \tag{2.17}$$

For known $\overline{f}(p,t_0)$, $\overline{f_\sigma}$, and $\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,t')\rangle$, Eqs. (2.4), (2.6), (2.16), and (2.17) form a closed set of integrodifferential equations for PDF $\overline{f}$ and average Green's function G. In Sec. III we further simplify Eqs. (2.16) and (2.17) by incorporating some approximations.

III. APPROXIMATE DIFFERENTIAL EQUATION

Under the condition when fluctuations in $\tilde{f}_\sigma$ are very rapid and correlated over a very short period of time, we can approximate Eqs. (2.16) and (2.17), written as

$$\begin{aligned}\langle \tilde{I}(q,t)\hat{G}(q,t;p,t')\rangle &= -\frac{\partial}{\partial q}\Bigg[\sum_\sigma \int\int e_\sigma^2\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\int_{t'}^{t} ds\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(q,t;p,t')\Bigg]\\ &\quad + \frac{\partial^2}{\partial q^2}\Bigg[\sum_\sigma \int\int \frac{e_\sigma^3}{2}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\int_{t'}^{t} ds\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(q,t;p,t')\Bigg],\end{aligned} \tag{3.1}$$

$$\begin{aligned}\langle \tilde{Q}(q,t)\hat{G}(q,t;p,t')\rangle &= -\frac{\partial}{\partial q}\Bigg[\sum_\sigma \int\int \frac{e_\sigma^3}{2}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\int_{t'}^{t} ds\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,s)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,t)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(q,t;p,t')\Bigg]\\ &\quad + \frac{\partial^2}{\partial q^2}\Bigg[\sum_\sigma \int\int \frac{e_\sigma^4}{4}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\int_{t'}^{t} ds\langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma G(q,t;p,t')\Bigg].\end{aligned} \tag{3.2}$$

Substituting Eqs. (3.1) and (3.2) in Eq. (2.6), changing t' to t_0, multiplying Eq. (2.6) by initial PDF $\overline{f(p,t_0)}$ and integrating over p, and using Eq. (2.4) we obtain an approximate differential equation for PDF $\overline{f}$, with $\lambda=1$, as

$$\frac{\partial}{\partial t}\overline{f}(q,t)+\frac{\partial}{\partial q}[\overline{I\,f}(q,t)]-\frac{\partial^2}{\partial q^2}[(\overline{Q}+D_1)\,\overline{f(q,t)}]$$
$$+2\frac{\partial^3}{\partial q^3}[D_2\overline{f}(q,t)]-\frac{\partial^4}{\partial q^4}[D_3\overline{f}(q,t)]=0, \qquad (3.3)$$

where

$$D_1(q,t)=\int_{t_0}^{t}\langle\widetilde{I}(q,t)\widetilde{I}(q,s)\rangle ds, \qquad (3.4)$$

$$D_2(q,t)=\int_{t_0}^{t}\langle\widetilde{I}(q,t)\widetilde{Q}(q,s)\rangle ds, \qquad (3.5)$$

$$D_3(q,t)=\int_{t_0}^{t}\langle\widetilde{Q}(q,t)\widetilde{Q}(q,s)\rangle ds. \qquad (3.6)$$

Equation (3.3) has the form of a Fokker-Planck equation with additional terms containing higher (third and fourth) order derivatives. Equations (1.7), (1.8), (2.13)–(2.15), and (3.3)–(3.6), with prescribed $\overline{f}_\sigma$ and $\langle\widetilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\widetilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle$, form a closed set of equations for charging of the dust particles in homogeneous dusty plasma. In principle, following Orszag and Kraichnan [27], equations for $\overline{f}_\sigma$ and $\langle\widetilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\widetilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle$ can be obtained by applying the DIA to the Vlasov equation after incorporating the terms which account for the interactions between plasma particles and dust particles. If the effects of dust particles on the plasma particles can be ignored, Maxwellian or any other appropriate distribution for plasma particles describing $\overline{f}_\sigma$ may be assumed as a first approximation. An expression for $\widetilde{I}$ can be approximately written in terms of the average value for $e_\sigma\gamma_\sigma v_\sigma$ and the plasma particles density fluctuation $\int\widetilde{f}_\sigma d\mathbf{v}_\sigma$,

$$\widetilde{I}=\sum_\sigma\int e_\sigma\gamma_\sigma v_\sigma\widetilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)d\mathbf{v}_\sigma$$
$$\cong\sum_\sigma\left[\frac{\int e_\sigma\gamma_\sigma v_\sigma\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)d\mathbf{v}_\sigma}{\int\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)d\mathbf{v}_\sigma}\right]\int\widetilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)d\mathbf{v}_\sigma, \qquad (3.7)$$

where the term inside the square brackets is the average value. A similar approximate expression can be written for $\widetilde{Q}$. Incorporating these approximations into Eqs. (3.4)–(3.6) allows us to write the expressions in terms of the density fluctuation correlation functions $[\langle\widetilde{n}_\sigma(t)\widetilde{n}_\sigma(s)\rangle]$ as

$$D_1(q,t)=\sum_\sigma\frac{\int\int e_\sigma^2\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma)\overline{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma)d\mathbf{v}_\sigma d\mathbf{v}'_\sigma}{\int\int\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma)\overline{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma)d\mathbf{v}_\sigma d\mathbf{v}'_\sigma}\int_{t_0}^{t}\langle\widetilde{n}_\sigma(t)\widetilde{n}_\sigma(s)\rangle ds, \qquad (3.8)$$

$$D_2(q,t)=\sum_\sigma\frac{\int\int\frac{e_\sigma^3}{2}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma)\overline{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma)d\mathbf{v}_\sigma d\mathbf{v}'_\sigma}{\int\int\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma)\overline{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma)d\mathbf{v}_\sigma d\mathbf{v}'_\sigma}\int_{t_0}^{t}\langle\widetilde{n}_\sigma(t)\widetilde{n}_\sigma(s)\rangle ds, \qquad (3.9)$$

$$D_3(q,t)=\sum_\sigma\frac{\int\int\frac{e_\sigma^4}{4}\gamma_\sigma(q,v_\sigma)\gamma_\sigma(q,v'_\sigma)v_\sigma v'_\sigma\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma)\overline{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma)d\mathbf{v}_\sigma d\mathbf{v}'_\sigma}{\int\int\overline{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma)\overline{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma)d\mathbf{v}_\sigma d\mathbf{v}'_\sigma}\int_{t_0}^{t}\langle\widetilde{n}_\sigma(t)\widetilde{n}_\sigma(s)\rangle ds, \qquad (3.10)$$

where $\tilde{n}_\sigma(t)$ is fluctuation over the mean density $(\overline{n_\sigma})$ of plasma particles. While writing Eqs. (3.8)–(3.10) we have assumed $\overline{f_\sigma}$ to be stationary, and have used

$$\overline{n_\sigma}=\int \overline{f_\sigma}(\mathbf{r},\mathbf{v}_\sigma)d\mathbf{v}_\sigma,$$

$$\int\int \langle \tilde{f}_\sigma(\mathbf{r},\mathbf{v}_\sigma,t)\tilde{f}_\sigma(\mathbf{r},\mathbf{v}'_\sigma,s)\rangle d\mathbf{v}_\sigma d\mathbf{v}'_\sigma=\langle \tilde{n}_\sigma(t)\tilde{n}_\sigma(s)\rangle. \tag{3.11}$$

Assuming the plasma to have stationary statistical properties for plasma particles, and considering an exponential form for the density correlation function with an integral time scale τ_σ, we write an approximate expression as

$$\int_0^t \langle \tilde{n}_\sigma(t)\tilde{n}_\sigma(s)\rangle ds \cong \int_0^t \langle \tilde{n}_\sigma(t)\tilde{n}_\sigma(t)\rangle e^{-(t-s)/\tau_\sigma}ds$$
$$=C_\sigma[\overline{n_\sigma}(t_0)]^2\tau_\sigma(1-e^{-t/\tau_\sigma}), \tag{3.12}$$

where $C_\sigma=\langle \tilde{n}_\sigma(t)\tilde{n}_\sigma(t)\rangle/[\overline{n_\sigma}(t_0)]^2$.

IV. MACROSCOPIC EQUATIONS

In this section we first present macroscopic equations [26] for the general case of homogeneous plasma. Then we present their numerical predictions for the temporal evolution of the mean and higher order moments of the charge of the dust particles for the particular case of a Maxwellian plasma. We define

$$\overline{q}(t)=\int q\overline{f}(q,t)dq, \quad S_n=\int (q-\overline{q})^n\overline{f}(q,t)dq, \tag{4.1}$$

where $\overline{q}$ is the average value of the charge on the particles, and S_n represents the nth moment of fluctuations in q over its mean value $\overline{q}$. Using these definitions and Eq. (3.3), we obtain approximate macroscopic equations [26] for $\overline{q}$, S_2, S_3, and S_4, written as

$$\frac{d\overline{q}(t)}{dt}-\overline{I}(\overline{q})-0.5S_2\overline{I}^{(2)}(\overline{q})=0, \tag{4.2}$$

$$\frac{dS_2(t)}{dt}-2S_2\overline{I}^{(1)}(\overline{q})$$
$$-2[\overline{Q}(\overline{q})+D_1(\overline{q},t)+0.5(\overline{Q}^{(2)}+D_1^{(2)})S_2]=0, \tag{4.3}$$

$$\frac{dS_3(t)}{dt}-3S_3\overline{I}^{(1)}-6(\overline{Q}^{(1)}+D_1^{(1)})S_2$$
$$-12[D_2(\overline{q},t)+0.5D_2^{(2)}S_2]=0, \tag{4.4}$$

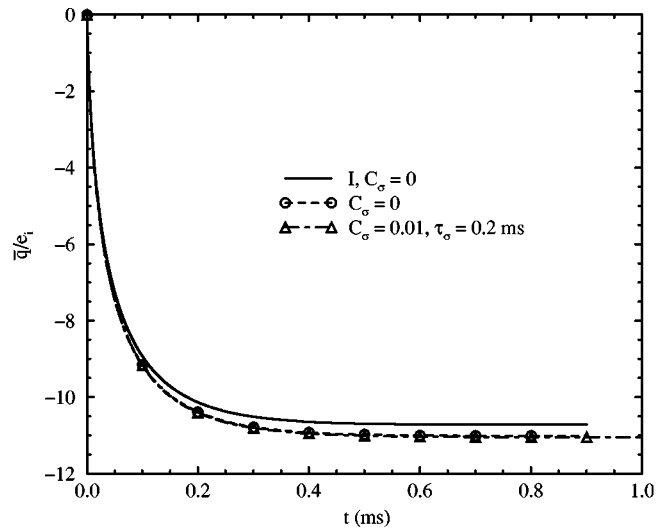

FIG. 1. Temporal evolution of the normalized mean dust charge $\overline{q}/e_i$.

$$\frac{dS_4(t)}{dt}-4I^{(1)}S_4-12[\{\overline{Q}(\overline{q})+D_1(\overline{q})\}S_2+(\overline{Q}^{(1)}+D_1^{(1)})S_3]$$
$$-48[D_2^{(1)}S_2+0.5D_2^{(2)}S_3]-24[D_3(\overline{q})+0.5D_3^{(2)}S_2]=0. \tag{4.5}$$

Here the superscript $n=1,2,\ldots$ represents the nth derivative of the function with respect to q and is evaluated at $q=\overline{q}$. In general, $\overline{I}$, $\overline{Q}$, D_1, D_2, and D_3 are nonlinear functions, and while writing Eqs. (4.2)–(4.5) we have used the Taylor series expansion for any nonlinear function F as

$$F(q)=F(\overline{q})+(q-\overline{q})F^{(1)}(\overline{q})+\frac{1}{2}(q-\overline{q})^2F^{(2)}(\overline{q})+\cdots. \tag{4.6}$$

The macroscopic equations (4.2) and (4.3) are now computed for a Maxwellian plasma (i.e., using the Maxwell distribution function for $\overline{f_\sigma}$) by a fourth-order accurate Runge-Kutta method with initial conditions $\overline{q}=S_2=0$ at time $t=0$. The typical values for various parameters used in the computation are the temperature of the ion $T_i=300$ K; the temperature of the electron $T_e=20T_i$; $M_e/M_i=1.4\times10^{-5}$, where M_i and M_e are the mass of the ion and the electron, respectively; the average number density of the electron, $\overline{n_e}=3.5\times10^{15}/m^3$; the average density of the ion, $\overline{n_i}=\overline{n_e}$; and $e_i=-e_e$. Using these values, computations are carried out for different values of the remaining parameters (C_σ,τ_σ) to investigate the effect of the fluctuations-fluctuation interactions, which are due to the plasma particle density fluctuations, on the charging behavior of the dust particles of radius 10 nm. We consider cases in which the root mean square density fluctuations for the plasma particles are either 1% (i.e., $C_\sigma=0.0001$) or 10% (i.e., $C_\sigma=0.01$) and the integral time scale $\tau_\sigma=0.1$ or 0.2 ms, which are small compared to the time required to reach the steady mean dust charge.

In Fig. 1, we present the temporal evolution of the nor-

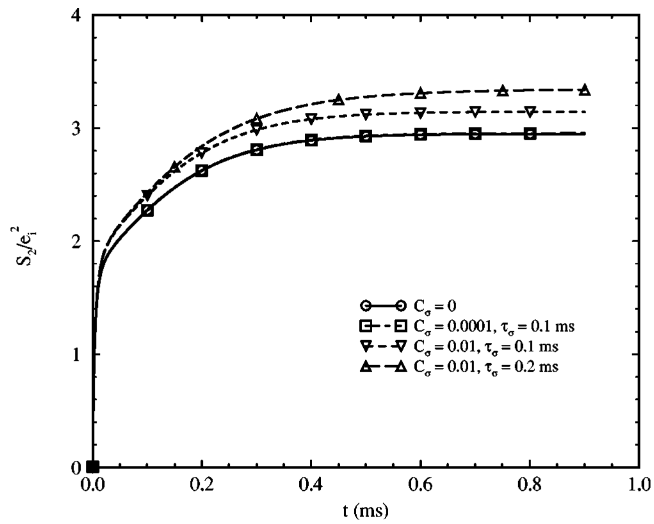

FIG. 2. Temporal evolution of the normalized variance S_2/e_i^2.

malized mean dust particle charge $\overline{q}/e_i$. In this figure, $C_\sigma=0$ refers to the cases in which the density fluctuations of plasma particles and consequently the fluctuations-fluctuation interactions are neglected. The first curve (denoted by I) with $C_\sigma=0$ is obtained by solving Eq. (4.2) while neglecting the term containing $\overline{I^{(2)}}$. This last term in Eq. (4.2) accounts for the contribution to the mean net current arising due to the dust particle charge distribution. The first term $\overline{I}(\overline{q})$ describes the net current when all the dust particles have a charge equal to $\overline{q}$. The comparison of curve I with the other curve for $C_\sigma=0$ shows the importance of the term containing $\overline{I^{(2)}}$, which is not zero in the Maxwellian plasma case as the expression for $\overline{I}$ is not a linear function of q and all the dust particles do not have the same charge due to the stochastic nature in the charging process. The inclusion of the fluctuations-fluctuation interactions in the computation by selecting nonzero values for C_σ and τ_σ does not change $\overline{q}$ appreciably when these values are small. This is reflected by the curve with $C_\sigma=0.01$.

The effect of the fluctuations-fluctuation interactions is more pronounced on the variance S_2, as shown in Fig. 2. In this figure, the temporal evolution of S_2/e_i^2 with and without the interaction terms are presented. It is observed that the variance increases from zero at time $t=0$, and asymptotically reaches a steady state value which can be obtained from Eq. (4.3) by setting $dS_2(t)/dt=0$ and using $\overline{q}$ at the steady state. At the steady state, the effect of the plasma particle density fluctuations appears in the computations through the value of $C_\sigma\tau_\sigma$, as suggested by Eqs. (3.12) when $t\to\infty$. According to the results in Fig. 2, the variance increases with the increase in the value of $C_\sigma\tau_\sigma$, i.e., with the increase in the magnitude and correlation of the plasma particle density fluctuations.

Using the steady state values of $\overline{q}$ for different values of C_σ, and τ_σ, the steady state values for S_2, S_3 and S_4 can be calculated from Eqs. (4.3), (4.4), and (4.5) by setting $dS_2(t)/dt=dS_3(t)/dt=dS_4(t)/dt=0$. In Table I, we present the steady state values for the normalized mean charge $\overline{q}/e_i$, the normalized variance S_2/e_i^2, the skewness ($S_3/S_2^{1.5}$), and the kurtosis (S_4/S_2^2) for different values of C_σ and τ_σ. It is clear from this table that as the value of $C_\sigma\tau_\sigma$ increases, the variance S_2 also increases whereas the skewness remains somewhat unchanged. Further, a noticeable decrease in the kurtosis is observed with the increase in $C_\sigma\tau_\sigma$. A comparison of the values of the kurtosis in Table I with its value for a Gaussian distribution (i.e., 3.0), suggests that the probability distribution function for the dust particle charge becomes increasingly more non-Gaussian with the increase in the intensity and/or the correlation time of the density fluctuations of the plasma particles.

V. CONCLUDING REMARKS

A probability density function modeling of charging of the dust particles was considered in homogeneous dusty plasma. The interactions between the phase space density of the plasma particles and the phase space density of dust particles were taken into account. The direct interaction approximation (DIA) method was used to tackle the closure problem, and an integrodifferential equation was obtained for the probability density function $\overline{f}(q,t)$ and the Green's function. These equations are further simplified to a differential form, and approximate expressions are suggested for various unclosed terms. The final PDF equation requires a prior knowledge of the ensemble average phase space density ($\overline{f}_\sigma$) for the plasma particles, as well as the coefficient C_σ and the integral time scale (τ_σ) appearing in Eq. (3.12) for computations. Though it is possible to solve the partial differential equation for PDF (3.3) by using the existing numerical methods and then to obtain the mean value for the charge ($\overline{q}$) and the higher moments of the charge fluctuations (S_n) for the dust particles, we also gave approximate equations govern-

TABLE I. Steady state values for the normalized mean dust charge ($\overline{q}/e_i$), the normalized variance (S_2/e_i^2), the skewness ($S_3/S_2^{1.5}$), and the kurtosis (S_4/S_2^2).

Case	$\overline{q}/e_i$	S_2/e_i^2	$S_3/S_2^{1.5}$	S_4/S_2^2
$C_\sigma=0$	-11.0070	2.948	9.35×10^{-2}	2.885
$\tau_\sigma=0.1$ ms, $C_\sigma=0.0001$	-11.0072	2.950	9.35×10^{-2}	2.884
$\tau_\sigma=0.2$ ms, $C_\sigma=0.0001$	-11.0074	2.952	9.35×10^{-2}	2.883
$\tau_\sigma=0.1$ ms, $C_\sigma=0.01$	-11.026	3.145	9.36×10^{-2}	2.801
$\tau_\sigma=0.2$ ms, $C_\sigma=0.01$	-11.044	3.346	9.38×10^{-2}	2.721

ing temporal evolution of $\overline{q}$ and S_n. These PDF and macroscopic equations can be solved numerically for Maxwellian and non-Maxwellian distributions for plasma particle energy, and a parametric study can be performed to assess the included effects of the fluctuations-fluctuation interactions and the involved approximations. As an example, the mean charge and the higher-order statistics S_n, for dust particles of radius 10 nm, have been computed for a Maxwellian plasma from the macroscopic equations. It is found that the fluctuations-fluctuation interactions enhance the variance, while decreasing the kurtosis and thus producing a non-Gaussian distribution for the dust particle charge. These results can be assessed against the data, which can be generated by the particle in cell simulation method [29] and/or by modifying the existing Monte Carlo simulation method due to Chunshi and Goree [10].

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